# Introductory Probability 20-09-07.

## Chapter 2 Axioms of probability

1. Introduction.

- Probability is a math area dealing with random behaviors.
- It has a history of more than 300 years in the study.
- It came from gambling in the early stage, and gamings of chance.

2. Random experiments, outcomes, sample space, events.

Random experiments / outcomes.

Example: ① Toss a coin to get a head or a tail.

② Roll a dice to see the number of the top face.

③ Measure the height of a randomly chosen student in the campus.

Def. (Sample space). The set of all possible outcomes of an experiment is called the sample space of the experiment.

Usually, We use $S$ to denote the sample space.

Example ① Toss a coin once.

$$S = \{H, T\}.$$

Toss a coin twice.

$$S = \{HH, HT, TH, TT\}$$

② Roll a dice once

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Roll a dice 3 times.

$$S = \{(i, j, k) : \quad i, j, k \in \{1, 2, 3, 4, 5, 6\}\}.$$

③ height of a randomly chosen student (in meters)

$$S = \{0 < x < \infty\} = (0, \infty)$$

Def (event) Let $S$ be the sample space of an experiment.

Every subset $E$ of $S$ is called an event.

If an outcome of the experiment is contained in the event $E$, then we say that $E$ has occured.

- Basic operations on events.

  Union: $E \cup F$

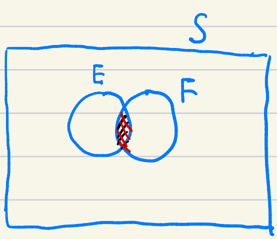
  Intersection: $E \cap F$

  Complement $E^c = S \setminus E$

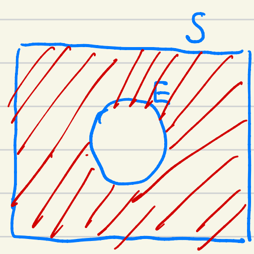
- $\emptyset$ Null event.

  We say two events $E, F$ are mutually exclusive
  if $E \cap F = \emptyset$.

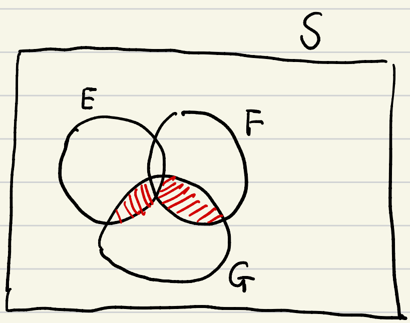
- Venn diagram:




$E \cap F$




$E^c$




$\left( (E \cap G) \cup (F \cap G) \right) \setminus (E \cap F \cap G)$

- Laws.

(i) $E \cup F = F \cup E, \quad E \cap F = F \cap E$ Commutative laws

$E \cap (F \cup G) = (E \cap F) \cup (E \cap G)$ distributive law

$$\left.\begin{aligned} E \cup (F \cup G) &= (E \cup F) \cup G \\ E \cap (F \cap G) &= (E \cap F) \cap G. \end{aligned}\right\}$$ associative laws

(ii) De Morgan's laws

$$\left( \bigcup_{n=1}^{\infty} E_n \right)^c = \bigcap_{n=1}^{\infty} E_n^c$$

$$\left( \bigcap_{n=1}^{\infty} E_n \right)^c = \bigcup_{n=1}^{\infty} E_n^c.$$

Pf. Let us prove the first equality in (ii)

$$x \in \left( \bigcup_{n=1}^{\infty} E_n \right)^c$$

$$\Leftrightarrow \quad x \in S, \quad x \notin \bigcup_{n=1}^{\infty} E_n$$

$$\Leftrightarrow \quad x \in S, \quad x \notin E_n \text{ for } n = 1, 2, \cdots$$

$$\Leftrightarrow \quad x \in E_n^c \text{ for } n = 1, 2, \cdots$$

$$\Leftrightarrow \quad x \in \bigcap_{n=1}^{\infty} E_n^c$$

Hence $\left( \bigcup_{n=1}^{\infty} E_n \right)^c = \bigcap_{n=1}^{\infty} E_n^c.$ ▨